The Dynamic Analysis of Investment System with Two Time Delays

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***Abstract*:** In order to explore the intrinsic relationship between investment corporations and investment projects, according to enterprise ecological theory, the investment corporations in accordance with the scope of investment projects is divided into small and medium-sized investment corporation (SMIC) and large investment corporations. According to enterprise ecosystem, the article builds a dynamic system model to explain the complex relationship between investment business and investment projects and we verify the results of the analysis by numerical simulation.

***Keyword*:** dynamic model; investment projects; investment corporations

1. Introduction

Recently, an increasing number of scholars applied ecological competition model theory to business and have made great achievements. On the basis of organizational ecology, Hannan[1] made a complete concept of organizational ecology and research framework and built a mathematical model which can weight the individual enterprise's development, change and succession. As the increasing fusion of the world economy and the economic environment increasing deterioration, Moore[2] proposes enterprise ecosystem evolution theory and believes corporations should beyond the perspective of the enterprise and develop its own evolution strategy in a view of business ecosystem point. Analysis of economic phenomena by differential equations has been widely used by some scholars, and recognized by an increasing number of economists. H.X.Yao[3] described an economic system by discrete-time equations, while X.L.Ke[4] and X.Z.He[5] can describe an asset pricing model truly by continuous-time equations.
The article is organized as follows. In section 2, we build an ecological model of investment corporations. We discuss the stability of the model in section 3, and some simulation examples are given to illustrate the obtained results. The last part, we state our results.

2. The model

Gaining investment projects is the mainly aim for an investment company. The relationship between investment projects and investment corporations is similar to the relationship between predator and prey in predator-prey model. Meanwhile, there are competitions among the investment corporations, which are similar to internal competitions among predators in the predator-prey model. So we can change the biological model, and use it to analysis investment activities. In order to study the relationship between of investment corporations and investment projects, this is a new idea, and a new method.

2.1 The basic assumption of the model

(1) A regional business groups (Investment Corporations or other corporations) can be seen as a biosphere in a population, assuming that the number of investment corporations per unit area can be accurately represent by a variable.

(2) In the investment market, investment corporations' scramble for resources is mainly reflected in the acquisition of the investment projects.

(3) We divide the investment corporations into small-medium investment corporations and large investment corporations according to the geographical scope of the projects which the investment corporations to bid for.

(4) Entrepreneurs are rational, before the investment company was established, and there is no competition in the same industry within a certain range.

(5) The entrepreneurs would not build investment corporations in a place until investment projects of the area have increased to a certain number.

2.2 Model and solution

We can get the investment corporation's ecological model by the stage-structure of predator-prey model proposed by Hasting[6]-[8].

1) investment projects

$$\frac{dx}{dt} = (a - \mu x)x - dx(z + y) \qquad (1)$$

$x(t)$, $y(t)$, $z(t)$ represent the density of investment projects, small-medium investment corporations, large investment corporations at time t at the unit area respectively. We use the Logistic equation proposed by Verhulst to explain the change of investment projects according to time t . The number of investment projects in a region not only has a natural expansion at the speed of ax, but also impacts by its own feedback at the rate of $-bx^2$. From the long term, it should meet the S-curve. a is the natural growth rate of investment projects. μ is the inhibition coefficient of investment projects. d is the effect coefficient between investment projects and investment corporations.

2) small and medium-sized investment corporation (SMIC)

$$\frac{dy}{dt} = cx\left(t-\tau_1\right)\left(z+y\right) - my^2 - eyz\left(t-\tau_2\right) \quad (2)$$

c is he growth rate of SMIC with the investment projects' increase. m means the competition among SMIC. e means the competition between SMIC and large investment corporations. $x(t-\tau_1)$ is the number of investment projects accumulated through time τ_1, $z(t-\tau_2)$ is the number of SMIC exist at $t-\tau_2$,which become large investment corporation through time τ_2.

3) large investment corporation

$$\frac{dz}{dt} = nyz\left(t-\tau_2\right) - bz^2 \quad (3)$$

n means the coefficient between SMIC and large investment corporations. b is the coefficient among the large investment corporations .(All coefficients are positive.)

We can get the dynamic system of investment from (1)(2)(3),as followed:

$$\begin{cases} \dfrac{dx}{dt} = (a-\mu x)x - dx(z+y) \\ \dfrac{dy}{dt} = cx\left(t-\tau_1\right)\left(z+y\right) - my^2 - eyz\left(t-\tau_2\right) \\ \dfrac{dz}{dt} = nyz\left(t-\tau_2\right) - bz^2 \end{cases} \quad (4)$$

System (4) has four equilibriums：

$$E_0 = (0,0,0),$$

$$E_1 = \left(\frac{a}{\mu}, 0, 0\right),$$

$$E_2 = \left(\frac{ma}{cd+\mu}, \frac{ca}{cd+\mu}, 0\right)$$

$$E_3 = \left(\frac{ab(ne+bm)}{cd(n+b)^2 + b\mu(ne+bm)}, \frac{abc(n+b)}{cd(n+b)^2 + b\mu(ne+bm)}, \frac{acn(n+b)}{cd(n+b)^2 + b\mu(ne+bm)}\right)$$

3. Analysis the properties of the system's solutions

From (4) we can get the jacobian determinant as followed

$$J = \begin{vmatrix} \lambda - a + 2\mu x + dy + dz & dx & dx \\ -c\left(y+z\right)e^{-\lambda\tau_1} & \lambda - cx + 2my + ez & -cx + eye^{-\lambda\tau_2} \\ 0 & -nz & \lambda + 2bz - nye^{-\lambda\tau_2} \end{vmatrix} \quad (5)$$

The characteristic equation is as followed for the equilibrium E_0

$$\lambda^2(\lambda - a) = 0$$

We can get the root of it

$\lambda_1 = a$, $\lambda_2 = \lambda_3 = 0$,so E_0 is unstable point. In an economic view, the number of investment projects in one region is zero, the numbers of the two types of investment corporations are zero, too. This phenomenon could not exist in our real life, so we do not study.

The characteristic equation is as followed for the equilibrium E_1

$$\lambda(\lambda + a)\left(\lambda - \frac{ac}{\mu}\right) = 0$$

We can get the root of it

$\lambda_1 = 0$, $\lambda_2 = -a$, $\lambda_3 = \dfrac{ac}{\mu}$.So E_1 is unstable point, too. Namely there are investment projects, but the numbers of small-medium investment corporations and large-invested enterprises are zero. Because the number of investment projects is increasing with time t, so the numbers of investment corporations would increase. Namely E_1 is unstable point.

The same to E_2,we can get the system exist eigenvalue $\lambda > 0$,so E_2 is unstable point. Namely, there are investment projects and SMIC, but the number of large investment corporation is zero. As the development of the social, a certain number of SMIC can meet market demand, when there are few investment projects. The SMIC will eventually develop into large investment corporations. So E_2 is unstable point.

For the positive equilibrium E_3, we can get the characteristic equation of E_3

$$P(\lambda) + Q(\lambda)e^{-\lambda\tau_1} + R(\lambda)e^{-\lambda\tau_2} + S(\lambda)e^{-\lambda(\tau_1+\tau_2)} = 0 \quad (6)$$

Let

$$\begin{aligned} P(\lambda) &= \lambda^3 + (2bz - cx + 2my + ez - a + 2\mu x + dy + dz)\lambda^2 \\ &+ \left[2bz(-cx+2my+ez) + (2bz - cx + 2my + ez)(-a + 2\mu x + dy + dz) - cxnz\right]\lambda \\ &+ (-a + 2\mu x + dy + dz)\left[2bz(-cx + 2my + ez) - ncxz\right] \\ &= \lambda^3 + p_2\lambda^2 + p_1\lambda + p_0 \end{aligned}$$

$Q(\lambda) = \left(\lambda + 2bz + nz\right)cdx(y+z) = q_1\lambda + q_0$,

$q_1 > 0$, $q_0 > 0$

$$\begin{aligned} R(\lambda) &= -ny\lambda^2 - ny(-cx + 2my + ez - a + 2\mu x + dy + dz)\lambda \\ &- ny(-a + 2\mu x + dy + dz)(-cx + 2my) \\ &= r_2\lambda^2 + r_1\lambda + r_0 \end{aligned}$$

$S(\lambda) = -cdnxy(y+z) = s_0$, $s_0 < 0$

3.1. $\tau_1 = \tau_2 = 0$

We can get for Eq.(6)

$$P(\lambda) + Q(\lambda) + R(\lambda) + S(\lambda) = 0 \quad (7)$$

$$\lambda^3 + (p_2 + r_2)\lambda^2 + (p_1 + q_1 + r_1)\lambda + (p_0 + q_0 + r_0 + s_0) = 0$$

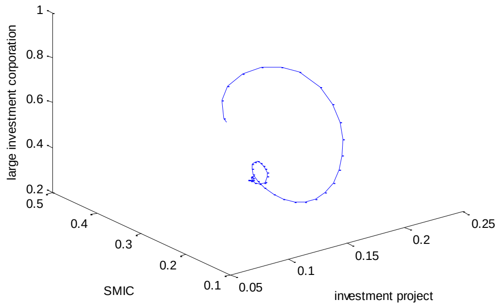
According to the Routh-Hurwitz criterion, the equilibrium point E_3 is stable if and only if

(a) $(p_2+r_2)>0$

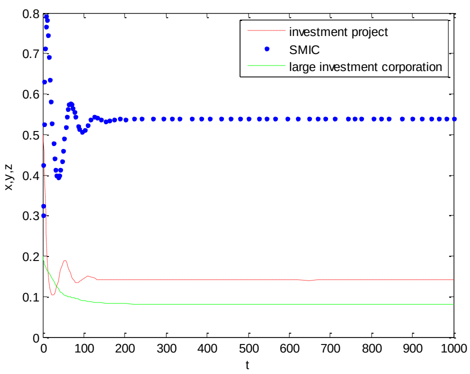
(b) $(p_2+r_2)(p_1+q_1+r_1)>(p_0+q_0+r_0+s_0)$

We set

a=0.2 , μ=0.1 , d=0.3 , c=0.4 , m=0.06 , e=0.4 , $n=0.03$, b=0.2 ,and $x=0.5$, $y=0.3$, $z=0.2$ .the number of investment projects and investment corporations reach a equilibrium point $E_3(0.1406, 0.5390, 0.0808)$,and simulation diagram shown as fig1(a-b)



(a)



(b)

Fig1(a-b) $\tau_1=\tau_2=0$, E_3 is asymptotically stable.

3.2. $\tau_1 \neq 0, \tau_2 = 0$

From Eq.(6), we can get

$$P(\lambda)+R(\lambda)+\left(S(\lambda)+Q(\lambda)\right)e^{-\lambda\tau_1}=0 \quad (8)$$

Namely

$$\lambda^3+(p_2+r_2)\lambda^2+(p_1+r_1)\lambda+(p_0+r_0)+(q_1\lambda+q_0+s_0)e^{-\lambda\tau_1}=0 \qquad (9)$$

Let $A_1=p_2+r_2$, $B_1=p_1+r_1$, $C_1=p_0+r_0$, $D_1=q_1$, $E_1=q_0+s_0$

We can get from (9)

$$\lambda^3+A_1\lambda^2+B_1\lambda+C_1+(D_1\lambda+E_1)e^{-\lambda\tau_1}=0 \qquad (10)$$

Lemma1.Eq. (8) has a unique pair of purely imaginary roots if $C_1^2<E_1^2$

Proof. If $\lambda=i\omega$, $\omega>0$ is a root of (10), separating real and imaginary parts, we have the following:

$$\begin{cases}\omega^3-B_1\omega=D_1\omega\cos\omega\tau_1-E_1\sin\omega\tau_1\\ A_1\omega^2-C_1=D_1\omega\sin\omega\tau_1+E_1\cos\omega\tau_1\end{cases} \qquad (11)$$

Squaring and adding both equations we have

$$\omega^6+Q_1\omega^4+Q_2\omega^2+Q_3=0 \qquad (12)$$

Where

$Q_1=A_1^2-2B_1$,

$Q_2=B_1^2-2A_1C_1-D_1^2$,

$Q_3=C_1^2-E_1^2$

We know that

$$Q_1=(2bz-ny)^2+(-cx+2my+ez)^2+(-a+2\mu x+dy+dz)^2+2cnxz>0$$

$$Q_3=C_1^2-E_1^2<0$$

Then the condition of this lemma implies that there is a unique positive root ω_0 satisfying (8). That is, (8) has a unique pair of purely imaginary roots $\pm i\omega_0$.

From (11) $\tau_{1,n}$ can be obtained

$$\tau_{1,n}=\frac{1}{\omega_0}\cos^{-1}\frac{D_1\omega_0^4+(A_1E_1-B_1D_1)\omega_0^2-E_1C_1}{D_1^2\omega_0^2+E_1^2}+\frac{2n\pi}{\omega_0}$$

And , $n=0,1,2,\cdots$

Lamma2. If the following conditions

$C_1^2<E_1^2$, $A_1(B_1+D_1)>C_1+E_1$,

$(B_1^2-2A_1C_1)E_1^2>C_1^2D_1^2$,

hold, system (4) undergoes Hopf bifurcation at E^* when $\tau_1=\tau_{1,n}$, $n=0,1,2,\cdots$; furthermore E^* is locally asymptotically stable if $\tau_1\in[0, \tau_{1,0})$; and unstable if $\tau_1>\tau_{1,0}$.

Proof. Differentiating (10) with respect τ_1, we get

$$\left[3\lambda^2+2A_1\lambda+B_1+D_1e^{-\lambda\tau_1}-\tau_1(D_1\lambda+E_1)e^{-\lambda\tau_1}\right]\frac{d\lambda}{d\tau_1}=\lambda(D_1\lambda+E_1)e^{-\lambda\tau_1}$$

That is

$$\left(\frac{d\lambda}{d\tau}\right)^{-1}=\frac{3\lambda^2+2A_1\lambda+B_1+D_1e^{-\lambda\tau_1}}{\lambda(D_1\lambda+E_1)e^{-\lambda\tau_1}}-\frac{\tau_1}{\lambda}$$

$$=-\frac{3\lambda^2+2A_1\lambda+B_1}{\lambda(\lambda^3+A_1\lambda^2+B_1\lambda+C_1)}+\frac{D_1}{\lambda(D_1\lambda+E_1)}-\frac{\tau_1}{\lambda}$$

Thus

$$\mathrm{Re}\left(\frac{d\lambda}{d\tau}\right)^{-1}\Bigg|_{\lambda=i\omega_0}=\mathrm{Re}\left(-\frac{B_1-3\omega_0^2+2A_1\omega_0 i}{i\omega_0\left[(C_1-A_1\omega_0^2)+i\omega_0(B_1-\omega_0^2)\right]}+\frac{D_1}{i\omega_0(D_1\omega_0 i+E_1)}\right)$$

$$=\frac{2D_1^2\omega_0^6+(3E_1^2+A_1^2D_1^2-2B_1D_1^2)\omega_0^4+(2A_1^2-4B_1)E_1^2\omega_0^2+(B_1^2E_1^2-C_1^2D_1^2-2A_1C_1E_1^2)}{\left[\omega_0^2(B_1-\omega_0^2)^2+(C_1-A_1\omega_0^2)^2\right]\left[(D_1\omega_0)^2+E_1^2\right]}$$

Let $M=\omega^2$

$$f(\omega)=2D_1^2\omega_0^6+(3E_1^2+A_1^2D_1^2-2B_1D_1^2)\omega_0^4$$
$$+(2A_1^2-4B_1)E_1^2\omega_0^2+(B_1^2E_1^2-C_1^2D_1^2-2A_1C_1E_1^2)$$

Then

$$G(M) = f(\omega)$$
$$= 2D_1^2M^3 + \left(3E_1^2 + A_1^2D_1^2 - 2B_1D_1^2\right)M^2$$
$$+\left(2A_1^2 - 4B_1\right)E_1^2M + \left(B_1^2E_1^2 - C_1^2D_1^2 - 2A_1C_1E_1^2\right)$$

And

$$G' = 2\left[3D_1^2M^2 + \left(3E_1^2 + A_1^2D_1^2 - 2B_1D_1^2\right)M + \left(A_1^2 - 2B_1\right)E_1^2\right]$$
$$\Delta = \left(3E_1^2 + A_1^2D_1^2 - 2B_1D_1^2\right)^2 - 12\left(A_1^2 - 2B_1\right)D_1^2E_1^2$$
$$= \left[3E_1^2 - \left(A_1^2 - 2B_1\right)D_1^2\right]^2 \geq 0$$

$G' = 0$ has two real roots, which take the form

$$M_1 = \frac{-\left(3E_1^2 + \left(A_1^2 - 2B_1\right)D_1^2\right) + \sqrt{\Delta}}{6D_1^2} < 0$$
$$M_2 = \frac{-\left(3E_1^2 + \left(A_1^2 - 2B_1\right)D_1^2\right) - \sqrt{\Delta}}{6D_0^2} < 0$$

From above, we can get that $G(M)$ monotonously increases in $(M_1, +\infty)$, which means that $f(\omega)$ monotonously increases in $(0, +\infty)$. And as we know

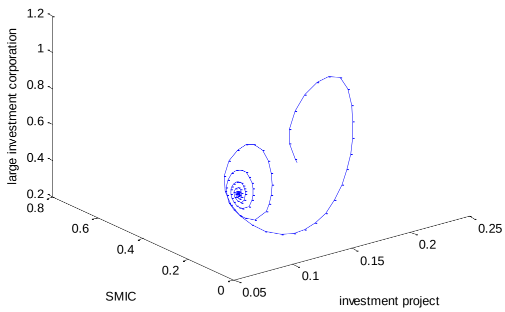
$$f(0) = B_1^2E_1^2 - C_1^2D_1^2 - 2A_1C_1E_1^2 > 0$$

We have $f(\omega) > 0$ for $\omega > 0$. Then we obtain

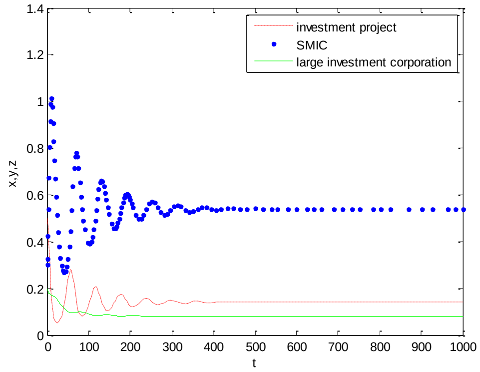
$$sign \left.\frac{d\left(\operatorname{Re}\lambda(\tau)\right)}{d\tau}\right|_{\tau=\tau_n} = sign \operatorname{Re}\left.\left(\frac{d\lambda}{d\tau}\right)^{-1}\right|_{\lambda=i\omega_0} > 0$$

While $\tau_{1,0}$ is the minimum τ_n at which the real parts of these roots are zero. So E_3 is locally asymptotically stable if $\tau_1 \in [0, \tau_{1,0})$ and unstable if $\tau > \tau_0$.

From above analysis, the system weather stability is decided by delay coefficient. If $\tau_1 \in [0, \tau_{1,0})$ E_3 is locally asymptotically stable. Namely, with the increase of investment projects, to some extent, the number of SMIC and large investment corporations is stability. See as fig2(c-d).If $\tau > \tau_0$, E_3 is unstable. The number of investment corporations has periodic change as the change of interment projects.,see as fig3(e-f).

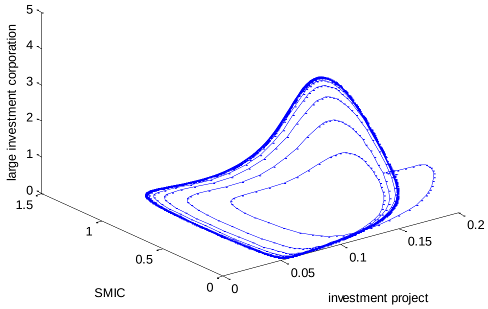


(c)

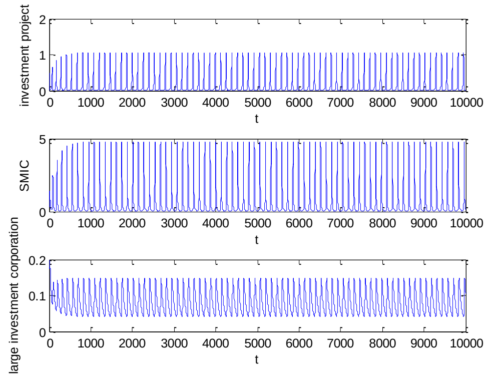


(d)

Fig2(c-d) $\tau_1 < \tau_{1,0}$, E_3 is asymptotically stable.



(e)

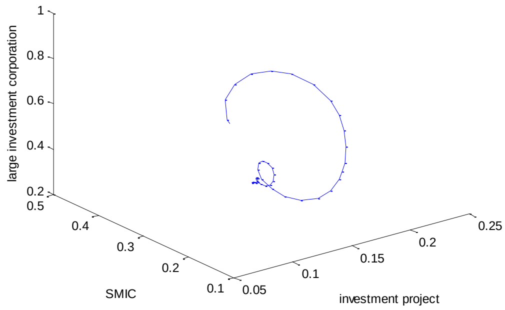


(f)

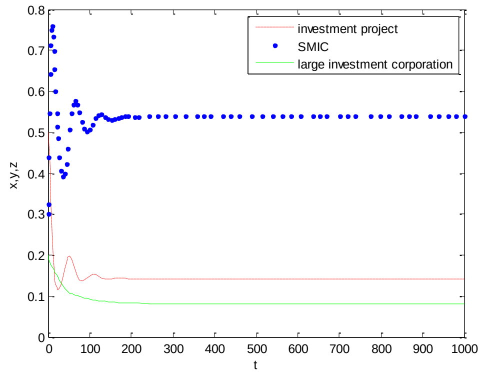
Fig3.(e-f) $\tau_1 > \tau_{1,0}$ Bifurcation periodic solution form the equilibrium point E_3 occur.

3.3. $\tau_2 \neq 0$

System simulation diagram is as follows



(g)



(h)

Fig4.(g-h） $\tau_1=0$, $\tau_2\neq 0$, E_3 is asymptotically stable.

We found that time delay τ_2 will not affect the stability of the system. The stability of the system has no matter with the accumulation of the time delay which the SIMC develop into the large investment corporations.

4. Summary

The article reveals the number of investment corporations with the change of the number of investment projects; through establish a model to analysis the relationship between investment corporations and investment projects. By adding time-delay to the investment model, we can find the market may rise to confusion for investment corporations entering market in different time. As time delay within the threshold, the system will reach the steady-state through a longer time. The system will have a period solution while time delay passes the threshold. What we study can provide a theoretical foundation for decision-makers choosing the time of entering the market, and provides some suggestions for managers to regulate the market better at macro point. The article doesn't take into account the complex relationship between investment and income, and we will consider it in the further research.

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